

The Quantum World

A New Kind of Physics

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- 1 Quantum Superposition
- 2 Probability and Tunneling
- 3 Quantum Statistics

What is a state?

- Particle: position and velocity
- Gas: pressure, volume, and temperature
- Light: Direction, intensity, polarization

Polarization

- Light is a combination of electric and magnetic fields
- The polarization direction is the direction of the electric field

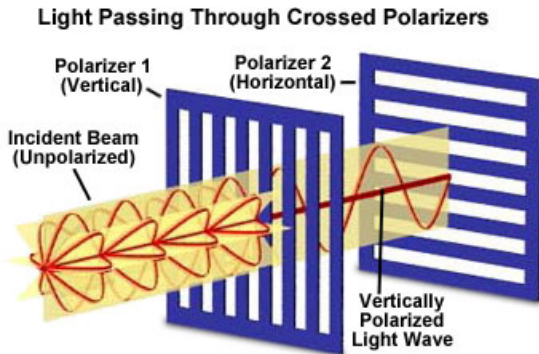
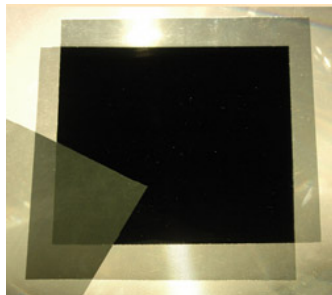


Figure 1

Measurement

- A polarization filter “measures” the polarization of light
- Two perpendicular filters block all light
- What about three filters?



Measurement Principle

- In quantum mechanics, measurements are active, not passive
- Measuring the polarization changed the polarization
- Is this true for other measurements?
 - Looking for something by shining a flashlight on it?
 - Checking speed using a radar gun?
 - Taking a temperature with a thermometer?

Uncertainty Principle

- If measuring a system changes its properties, are its properties well-defined?
- Heisenberg's uncertainty principle: there is some irreducible uncertainty

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Two Kinds of Uncertainty

- Local hidden variables: uncertainty because of incomplete knowledge
 - We measure temperature, but individual particle motion is hidden
- “True uncertainty”: the system itself is not in a determinate state
 - Particles do not have exact positions or velocities

- How can a system be in an indeterminate state?

Quantum States

- How can a system be in an indeterminate state?
- Answer: quantum states are superpositions of determinate states

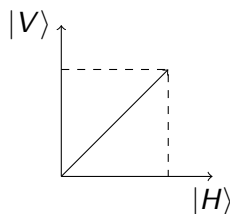
- How can a system be in an indeterminate state?
- Answer: quantum states are superpositions of determinate states
- Wave-particle duality: since particles are “spread out” across different position, they have some properties of waves

Example: Polarization

- A vertical polarization filter can measure two “definite” states, $|V\rangle$ and $|H\rangle$
- A quantum state of polarization looks like $\alpha|V\rangle + \beta|H\rangle$
- Measurement with a vertical filter: fraction α^2 passes through
- Measurement with a horizontal filter: fraction β^2 passes through

Diagonal Polarization

- We observe that diagonally polarized light has its intensity reduced by $\frac{1}{2}$ for each filter
- This means $\alpha^2 = \beta^2 = \frac{1}{2}$
- The diagonally polarized state is $\frac{1}{\sqrt{2}} |V\rangle + \frac{1}{\sqrt{2}} |H\rangle$

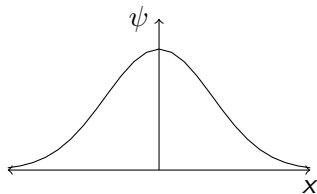


- With more than one particle, quantum states are joint superpositions
- Polarization example: $\alpha |V, V\rangle + \beta |H, V\rangle + \gamma |V, H\rangle + \delta |H, H\rangle$
- Born interpretation: think of $\alpha^2, \beta^2, \gamma^2, \delta^2$ as probabilities
- Two systems may be correlated (e.g., $\alpha = \delta = 0$) – this is called entanglement

- Instead of bits ($|V\rangle$ and $|H\rangle$), use qubits ($\alpha|V\rangle + \beta|H\rangle$)
- Entangled qubits give larger computational power than independent bits
 - Shor's algorithm: fast factorization of large integers
 - Grover's algorithm: search through N items in \sqrt{N} time

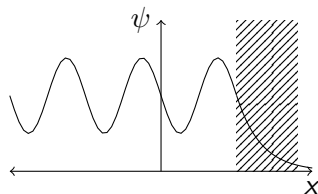
Wavefunctions

- With polarization, quantum states were superpositions of 2 states
- A position state is a superposition of infinitely many states
- We represent the state by $\psi(x)$, a function giving the “probability amplitude” at x



Waves at Boundaries

- Waves (classical and quantum) satisfy continuity conditions
- At a boundary they cannot cross, waves vanish exponentially, but not instantly
- Examples:
 - Light striking metal
 - Magnetic fields and superconductors



- If the barrier is finite, the wave is not completely suppressed
 - Classical example: a very thin sheet of metal may be translucent
- Quantum wavefunctions represent probabilities
- There is a finite probability of a particle “tunneling” through a classically insurmountable barrier
 - Example: α decay of heavy nuclei

Tunneling Electron Microscope

Pauli Exclusion Principle

- Electrons in atoms can't occupy the same quantum state
- Periodic table reflects this principle

Group → 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
↓ Period

1	1 H																2 He	
2	3 Li	4 Be										5 B	6 C	7 N	8 O	9 F	10 Ne	
3	11 Na	12 Mg										13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La	* 72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	* 89 Ac	* 104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				* 58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				* 90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

Fermions and Bosons

- All fermions obey the Pauli principle
- Bosons can occupy the same state
- Photons are bosons, electrons are fermions



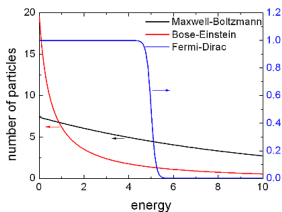
Spin Statistics Theorem

- Particles are classified as fermions or bosons by spin
- Half-spin particles are fermions, integer-spin particles are bosons

Fermions	Bosons
Electron	Photon
Proton	Gluon
Neutron	Pion

Occupation Functions

- Classical particles: Maxwell-Boltzmann distribution
- Fermions: Fermi-Dirac distribution
- Bosons: Bose-Einstein distribution



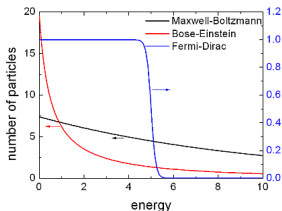
Bose-Einstein Condensate

Superconductors

- Electrons have spin $1/2$, so pairs of electrons (called Cooper pairs) can act as bosons
- Cooper pairing occurs at low temperatures in some materials
- When electrons act as bosons, current can flow like a laser

Degenerate Matter

- Stars are led to contract by gravity and led to expand by hydrostatic pressure
- After a star runs out of fuel to burn, hydrostatic pressure cannot balance gravity
- Stars contract until electrons have filled all available states
- Electron-degenerate matter forms a white dwarf



Chandrasekhar Limit

- Electron degeneracy pressure can be relieved if electrons enter nuclei
- $p + e \rightarrow n + \nu_e$ creates a neutron star
- The white dwarf must reach a critical mass for this to occur
- This leads to a Type 1a supernova

