Einstein's Dream

The Theory of Everything

Ross Dempsey

Department of Physics and Astronomy
Johns Hopkins University

JHU Splash, 2017

Outline

Euclidean Space

- Euclidean space is characterized by the Pythagorean theorem for distance:

$$d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

- In special relativity, distance is replaced with the spacetime interval:

$$s = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2}$$

- In either case, space (or spacetime) is "flat"

Curved Space

- How can we generalize the distance formula?
- Squeeze or stretch along axes, rotate coordinates:

$$d = \sqrt{g_{xx}\Delta x^2 + g_{xy}\Delta x \Delta y + \ldots + g_{zz}\Delta z^2}.$$

- This is only changing the coordinates – we haven't changed space

Curved Space

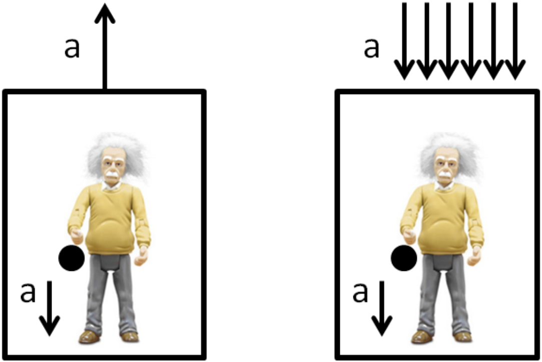
- Squeeze, stretch, and rotate in different ways at different places:

$$d = \sqrt{g_{xx}(x,y,z)\Delta x^2 + g_{xy}(x,y,z)\Delta x \Delta y + \ldots + g_{zz}(x,y,z)\Delta z^2}.$$

- Space becomes only *locally* Euclidean
- Mathematical theory developed by Riemann in 19^{th} century

Relativity and Gravity

- Relativity: all frames are equivalent
- Is Einstein accelerating up, or is he in a gravitational field?



Equivalence Principle

- A gravitational field is *equivalent* to a non-inertial frame of reference (e.g., an accelerating elevator)
- In special relativity, we work with inertial frames
- Inertial frames have the flat spacetime metric

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -c^2 \end{pmatrix}$$

- How do we model non-inertial frames?

Curved Spacetime

- Einstein generalized relativity to include curved spacetime geometries
- The metric g becomes a dynamical quantity
- How do we predict what g will be?

Einstein Field Equations

- The metric g is technically a second-rank tensor *field* – Einstein field equations tell us its behavior
- Important variables
  - $R_{\mu\nu}$ and R are related to g
  - Λ is the "cosmological constant"
  - G is Newton's gravitational constant
  - c is the speed of light
  - T describes the distribution of mass and energy in space
- Ten equations are hidden inside

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

Solutions to EFE

- Exact solutions: black holes
  - No-hair theorem: mass, charge, angular momentum
- Approximate solutions: gravitational waves
  - Moving perturbations in spacetime
  - Radiates energy similar to electromagnetic waves

LIGO

- Gravitational waves from the strongest sources have only very slight effects on Earth
- How can we detect them?

LIGO

- Gravitational waves from the strongest sources have only very slight effects on Earth
- How can we detect them?
- LIGO: Laser Interferometry Gravitational Observatory
- Essentially a Michelson interferometer

LIGO

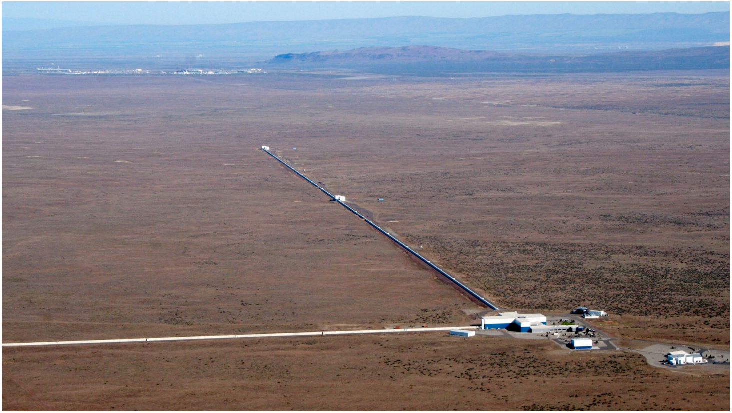


Figure: LIGO is a precise Michelson interferometer with 4 km arms.

Black Hole Merger

Quantizing a Field

- Quantum mechanics is based on classical mechanics: particles moving in potentials
- How does quantum theory apply to electromagnetism, a field theory?
- Fields become quantum particles

Quantum Harmonic Oscillator

- Common problem in quantum mechanics: mass on a spring
- Result is evenly spaced energy levels: $E_n = \hbar\omega \left(n + \frac{1}{2}\right)$
- One possible solution uses "creation" and "annihilation" operators:

$$\hat{a} \left|n\right\rangle \propto \left|n-1\right\rangle$$
$$\hat{a}^\dagger \left|n\right\rangle \propto \left|n+1\right\rangle$$

Fields as Oscillators

- Fields can be "excited" at different frequencies
- At each frequency, equations take the form of a quantum harmonic oscillator
- We treat fields as collections of harmonic oscillators, with creation and annihilation operators corresponding to each frequency
- In the electromagnetic case, these operators correspond to photons

Divergent Energy

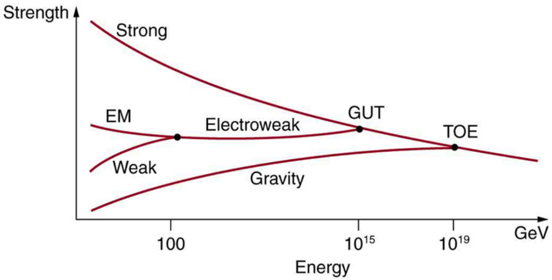
- Recall the energy of a harmonic oscillator: $E_n = \hbar\omega \left(n + \boxed{\frac{1}{2}} \right)$
- An infinite collection of harmonic oscillators has infinite energy
- Common problem in quantum field theory – solved by *renormalization*

Experimental Precision

- When the mathematical issues are properly solved, QFT is extremely accurate
- Example: anomalous magnetic dipole moment
  - Experiment: .0011596521808
  - Theory: 0.001159652181643

Grand Unified Theories

- It is known that electromagnetism and the weak force are both artifacts of an electroweak force
- Grand unified theories include the strong force as well
- What about gravity?



Theory of Everything

- A so-called "theory of everything" must incorporate gravity as well
- General relativity and quantum field theory are based on different principles and are hard to reconcile
- Many physicists are searching for a quantum theory of gravity

Quantum Gravity

String Theory

- One possible theory of quantum gravity is string theory
- Postulates that particles are actually extremely small strings
- Vibrational modes of strings correspond to types of particles

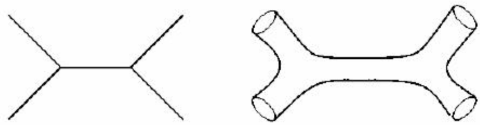


Fig.3: Particle scattering processes (left), string scattering processes (right).

Compactified Dimensions

- String theory requires extra dimensions of space
- Small finite dimensions would only be detectable at very high energies

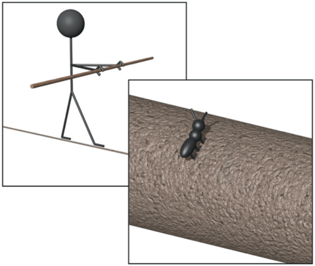
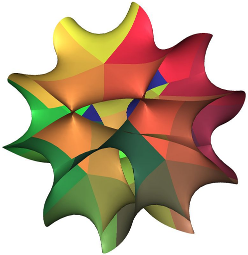


Figure 2: Example of a compact dimension

Calabi-Yau Space

- One possibility for extra dimensions is a Calabi-Yau manifold
- A Calabi-Yau manifold with three holes would lead to three families of particles



Discussion